

CNS Institute for Physics Teachers

Title:	Resonance in Transverse Waves
Original:	1 July 2004
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Appropriate Level:	Regents Physics
Abstract:	This experiment is designed to develop the student's understanding of resonance. Students explore the conditions needed to establish standing waves on a string and discover the relationship between nodes and antinodes for different resonant modes. Students also determine the relationship between frequency and wavelength for a fixed length of string.
Time Required:	One 40-minute period for lab activities. If experimental design and post-lab analysis are done in class, two more class periods are needed.
NY Standards Met:	4.3c The model of a wave incorporates the characteristics of amplitude, wavelength, frequency, period, wave speed, and phase. 4.3f Resonance occurs when energy is transferred to a system at its natural frequency. 4.3m When waves of a similar nature meet, the resulting interference may be explained by the principle of superposition. Standing waves are a special case of interference.
Special Notes:	Resonance is a kit available through the CIPT Equipment Lending Library, www.cns.cornell.edu/cipt/ .

Behavioral Objectives:

Upon completion of this lab activity, a student should be able to:

- Explain that standing waves only occur at special frequencies called resonant frequencies.
- Be able to use the wave equation $v = f\lambda$ to solve for the velocity of a standing wave in a fixed-length string.
- Explain the relationship between the nodes and antinodes of a standing wave and its wavelength.

Class Time Required:

One 40-minute class period. If analysis and experimental design are done in class, two more class periods are needed.

Teacher Preparation Time:

15 minutes

Materials Needed:

The equipment listed below is available from the CIPT Equipment Lending Library as the Resonance kit. Visit www.cns.cornell.edu/cipt/ for to reserve the kit.

Each student group needs:

- Venire Lab Pro Interface
- TI-83 Plus calculator **or**
- Vernier LabQuest
- Pendulum clamp
- Connector cable
- Resonator unit with power supply
- Ring stand with cross bar

Assumed Prior Knowledge of Students:

Wave equation $v = f\lambda$

Constructive and destructive interference

Answers to Questions:**Standing Waves**

1. Look around. Are all the string lengths that form a half-wave the same length? No.
2. Other than length, how is your system different from other systems in the room? The frequencies are different.
3. Change the frequency of vibration of the motor unit by adjusting the speed control. What happens to your half-wave? It disappears.
4. Predict how you can re-form your half-wave without changing the vibration rate of the motor unit. Answers will vary. The correct prediction is to adjust the length of the string.

5. Test your prediction. What did you do to re-form your half-wave? Adjusted the length of the string.
6. The wave you have been observing is called a “standing wave.” Why do you think this term was chosen? The wave pattern appears to stand still and not travel along the string.
7. When a system is driven at a special frequency called its “natural frequency,” it has a large amplitude response. This is known as “resonance.” Are standing waves an example of resonance? Explain. Standing waves are an example of resonance because when the motor vibrates at the right frequency, the string oscillates with a large amplitude in a standing wave pattern.

Nodes and Antinodes

1. If *three* nodes produce a complete wavelength as you see here, what is the distance between *two* nodes in terms of wavelength? 1/2 wavelength
2. How many antinodes are there with three nodes? 2 Two nodes? 1
3. How many wavelengths fit on a string with *four* nodes? 1.5 wavelength
4. Have your partner grab the node in the middle. Describe what happens to the portion of the standing wave below his or her fingers. Holding the string at the node has little or no effect on the standing wave.
5. Is it possible to get a standing wave with 1/4 or 3/4 of a wave-length on the string? Explain. No. The ends of the string must always be nodes because they are fixed points. Therefore, the standing wave must always be an integer multiple of a 1/2 wavelength.

Resonant modes of a fixed-length string

Experimental Design:

1. List all the variables (physical quantities that can change) in the motor/string system. A basic list includes length of the string, wavelength of the waves, frequency (or period) of the motor.
2. Which of the variables you listed in 1. will you keep constant in this experiment? The length of the string will be kept constant.
3. Which of the variables you listed in 1. will you change, or allow to change? The frequency and the wavelength will change.
4. Given a photogate¹, a meterstick, and anything else in the room, how will you measure each of the variables? The photogate can be used to measure the period of the string motion. The inverse of the period is the frequency. A meterstick can be used to measure the length of string. When divided by the number of half wavelengths in the string, this gives the wavelength.
5. Design a procedure aimed at answering the Question, using what you wrote in 1. – 4. Hang the motor from the string and measure the length of the string. While keeping the length of

¹ A photogate is an electronic timer containing a light beam. Every time an object blocks the light beam, the timer starts or stops.

string fixed, adjust the frequency of the motor through its full range. Stop whenever a standing wave occurs, measure the period of vibration with the photogate, and calculate the wavelength from the number of half-wavelengths on the string.

Data table – sample data given below

Overall length of string from suspension point to motor unit = 0.829 m

Number of nodes	Distance between adjacent nodes (m)	Wavelength (m)	Period of vibration (s)	Standard deviation of period values (s)	Frequency (Hz)	Wave speed in the medium (the string) (m/s)
2	<u>0.829</u>	<u>1.66</u>	<u>0.0448</u>	<u>0.0002</u>	<u>22.3</u>	<u>37.1</u>
3	<u>0.415</u>	<u>0.829</u>	<u>0.0223</u>	<u>0.0000</u>	<u>44.8</u>	<u>37.2</u>
4	<u>0.276</u>	<u>0.553</u>	<u>0.0147</u>	<u>0.0001</u>	<u>68.0</u>	<u>37.6</u>
5	<u>0.207</u>	<u>0.415</u>	<u>0.0109</u>	<u>0.0001</u>	<u>91.7</u>	<u>38.1</u>

Post-lab Analysis

1. You measured the period of the oscillation and the distance between nodes. Calculate the frequency and the wavelength for each resonance from this information, and record the results in the data table. See data table.
2. Look at the frequency column in your data table. How are the frequencies of the different resonant modes related to the frequency of the first (fundamental) mode? The frequencies are approximately integer multiples of the lowest frequency.
3. If you were to make a standing wave with 6 nodes, predict the frequency that would produce this resonance, assuming the same length of string. Show your work. It would be approximately 5 times the fundamental frequency or $5 \times (22.3 \text{ Hz}) = 112 \text{ Hz}$.
4. Look at the wavelength column in your data table. How are the wavelengths of the different resonant modes related to the wavelength of the fundamental mode? The wavelengths of the resonant modes are equal to the wavelength of the fundamental mode divided by the number of half wavelengths in the string.
5. If you were to make a standing wave with 6 nodes, predict the wavelength of this resonance, assuming the same length of string. Show your work. 6 nodes are 2.5 cycles and measure 2.5 wavelengths. $2.5\lambda = (0.829 \text{ m})$ and $\lambda = 0.332 \text{ m}$. OR $\lambda = 2(0.829)/5 = 0.332 \text{ m}$.
6. Your motor can only explore a limited range of frequencies. If you had an ideal motor that could vibrate at any frequency, would you expect to find any additional resonant modes at lower frequencies? At higher frequencies? Explain your answer. There would not be any additional resonant modes at lower frequencies because the string needs a minimum of two nodes, one at each end. This mode has been measured. There would be many modes at higher frequencies because the string can vibrate with a virtually infinite number of nodes.
7. Use $v = f\lambda$ to fill in the remaining column in the data table. Use those values to make your best estimate of v , the speed of the waves on the string. Averaging the values for v in the data table gives a velocity of 37.5 m/s.

8. Does the velocity of the waves change significantly or remain essentially constant? Do you expect the velocity to change? Explain your reasoning. The velocity remains essentially constant. This is expected because the velocity is determined by the properties of the medium, which remain unchanged.
9. From your values for v , predict the length of string that would resonate at 75 Hz with two nodes. Explain your reasoning. Using $v = f\lambda$, the wavelength would be $\lambda = v/f = (37.5 \text{ m/s})/(75 \text{ Hz}) = 0.50 \text{ m}$. For two nodes, the string would be a half wavelength long, which is $(0.50 \text{ m})/2 = 0.25 \text{ m}$.
10. You learned in this lab that the wave speed in the string stayed constant. With the help of the following equation, list two ways you could increase the wave speed of a string.

$$v = f\lambda = \sqrt{\frac{\tau}{\mu}} \quad \text{where } \tau = \text{tension of the string and } \mu = \text{mass/unit length of the string}$$

You could increase the tension of the string by hanging more weight from it and you could decrease the mass/unit length by using finer string.

Tips for the Teacher:

- It is highly recommended to use the power supply with the LabPro, due to its high consumption of power. If battery power must be used, use the following procedure to avoid wasting batteries: when done using the LabPro disconnect the photogate, quit the DataGate software, and turn off the calculator. The best way to ensure that the LabPro is off is to remove one of the batteries. Even when all devices appear to be turned off, the batteries may still drain.
- If the motor is turned on but not rotating, move the speed control knob to the maximum speed. The motor will usually start spinning. Then you can adjust the motor to the desired speed.
- Remind students to turn off the motors when not in use.
- If your students have not used a photogate before, you will want to explain how it works. The photogate has an infrared light source and an infrared detector separated by an open space. As the string passes through that space, the gate's light path is interrupted. The small red LED on the photogate will light to indicate that the beam is being interrupted by the string. The LabPro or LabQuest determines the time between passes, averages these for 10 oscillations, and returns the period of vibration to the nearest .001 second.
- When the students are using the photogates, emphasize that the infrared beam of the photogate should be centered on the motion of the string. If the beam is positioned on the edge of the string's motion, the photogate may report twice the actual period.
- You may also want to remind your students about standard deviation. The photogate returns the standard deviation for the 10 period measurements. If the standard deviation is more than $\sim 1 \times 10^{-4}$ in this experiment, then the trial should not be recorded. Standard deviation is a measure of the spread of data, and a large spread indicates a likely error in the measurement. Errors can be caused by a swinging motor or a photogate that is not centered on the motion of

the string. Emphasize that if the standard deviation is too high, students should try to steady the motor unit, and/or adjust the position of the photogate; then run the trial again.

- It may be useful to do a review of the wave equation with students. Below is a derivation of the wave equation.
 - For motion with a constant velocity v , one uses $x = vt$ where x is the distance and t is the time traveled.
 - The speed of a wave is constant within a given medium, and one cycle of a wave will move its wavelength ($\lambda =$ the distance for one cycle) in its period ($\mathbf{T} =$ the time for one cycle), making the simple motion equation applied to a wave become: $\lambda = \mathbf{vT}$
 - Recall that period and frequency are reciprocals of each other. Replace \mathbf{T} in the last equation with $1/\mathbf{f}$, and rearrange the equation. It becomes the familiar $\mathbf{v} = \mathbf{f}\lambda$

Equipment List

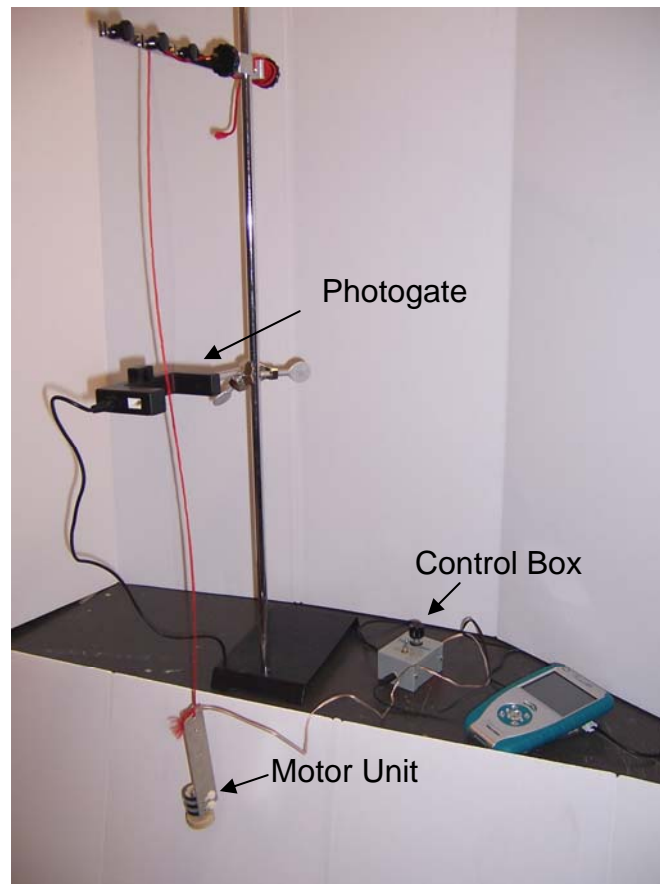


Photo ID	No.	Item
1	1	Vernier Lab Pro Interface ***
2	1	TI-83 Plus calculator
3	1	Pendulum clamp
4	1	Connector cable
5	1	Vernier Photogate
6	1	Vernier LabQuest ***
7	1	Resonator unit
Not in picture		Ring stand with cross bar

***** Note – You need either the Vernier Lab Pro Interface and TI-83 Plus Calculator or the Vernier LabQuest, but not both to complete this lab.**

RESONANCE IN TRANSVERSE WAVES

Waves come in two main varieties: transverse and longitudinal. When you drop a pebble in a lake, the ripples spread outward along the surface, while a blade of grass on the surface bobs up and down. The water molecules oscillate up and down, moving vertically between extreme positions, and carrying the blade of grass up and down with them. Meanwhile, the wave propagates (moves) horizontally out along the surface. Since the motion of the water molecules is perpendicular to the propagation direction of the wave (and the direction of energy flow), such waves are called *transverse* waves. (The prefix ‘trans’ means ‘across’). In contrast, the displacement is parallel to wave propagation axis for longitudinal waves. In this lab you will explore transverse waves. As you will see, a wave on a string is a transverse wave, since the string oscillates perpendicular to its axis.

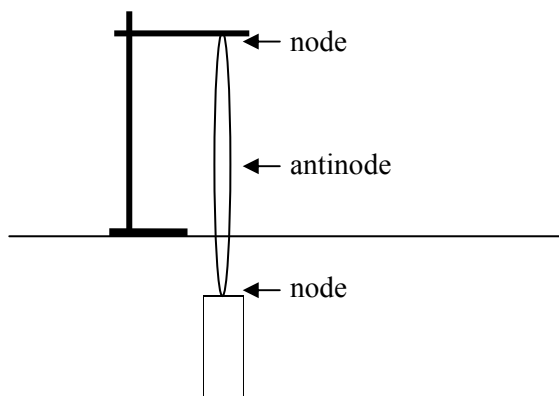


Standing Waves

- Loop the string of the motor unit over the arm of the pendulum clamp to obtain the longest string length possible.
 - Set the control box on a chair or bench nearby so that the wire hangs loosely draped between the motor unit and the control box. It is ok for the control box wires to touch the floor.
 - Turn on the motor and set it to a medium to slow speed. Pull the free end of the string to slowly raise the motor unit. Stop pulling and secure the string when a half-wave appears in the string.
1. Look around. Are all the string lengths that form a half-wave the same length? _____
 2. Other than length, how is your system different from other systems in the room?
 3. Change the frequency of vibration of the motor unit by adjusting the speed control. What happens to your half-wave?
 4. Predict how you can re-form your half-wave without changing the vibration rate of the motor unit.
 5. Test your prediction. What did you do to re-form your half-wave?
 6. The wave you have been observing is called a “standing wave.” Why do you think this term was chosen?
 7. When a system is driven at a special frequency, called its “natural frequency,” it has a large amplitude response. This is known as “resonance.” Are standing waves an example of resonance? Explain.

Nodes and Antinodes

Although a standing wave appears to stand still, it is actually made from traveling waves moving in opposite directions along the string. When there is a standing wave, complete destructive interference occurs only at certain locations called nodes, where the medium barely moves at all. Maximum constructive interference occurs at locations called antinodes, where the medium oscillates with maximum amplitude. The simplest standing waves in the string have two nodes, one at each end, as in the diagram below.



- Clamp the string to the stand so that it hangs almost to the floor, and set the speed of the motor to roughly $2/3$ of full speed.
 - Pinch the string between your thumb and forefinger to create a ‘finger-clamp’ on the string. Slide your finger-clamp down the string until you create a standing wave.
 - By changing the effective string length in this fashion, form a standing wave with only three nodes. Note how this resembles a full cycle of a wave, or one wavelength.
1. If *three* nodes produce a complete wavelength as you see here, what is the distance between *two* nodes in terms of wavelength? _____ wavelength
 2. How many antinodes are there with three nodes? _____ Two nodes? _____
 3. How many wavelengths fit on a string with *four* nodes? _____ wavelength
 4. Have your partner grab the node in the middle. Describe what happens to the portion of the standing wave below his or her fingers.
5. Is it possible to get a standing wave with $1/4$ or $3/4$ of a wave-length on the string? Explain.
- Turn off the motor.

Resonant modes of a fixed-length string

By now, you will have noticed that resonance (standing waves) only occurs for special conditions. You will determine what these special conditions are for a fixed-length string. In the first part of the lab, you changed the length of the string. However, it often happens that the wave medium (in this case, the string) is a fixed length.

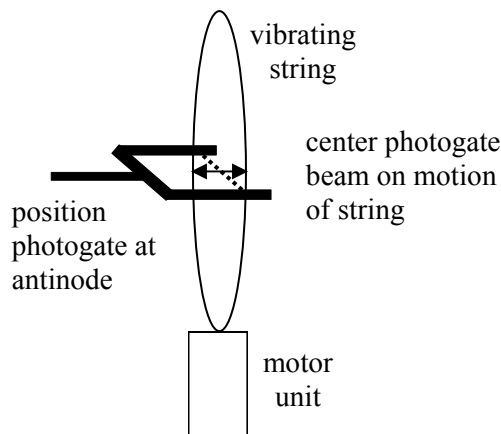
Question: What are the conditions that create standing waves (resonances) on a fixed-length string?

Experimental Design:

1. List all the variables (physical quantities that can change) in the motor/string system.
2. Which of the variables you listed in 1. will you keep constant in this experiment?
3. Which of the variables you listed in 1. will you change, or allow to change?
4. Given a photogate², a meterstick, and anything else in the room, how will you measure each of the variables?
5. Design a procedure aimed at answering the Question above, using what you wrote in 1. – 4.

As you follow the steps below, note the similarities and differences between this procedure and the one you proposed.

- Turn on the motor and set it to as low a speed as practical.
- Pull the free end of the string to slowly raise the motor unit. Stop pulling and secure the string when the largest amplitude, half cycle standing wave (2 nodes) appears in the string.
- Measure the string length between the nodes and record it in the data table on page .
- To measure the period of the wave, place the photogate as close to an antinode as possible (where the string moves most). Make sure that the motor unit is hanging without swinging. Then carefully center the beam of the photogate on the motion of the string.
- Using the appropriate photogate instructions in the box below, measure the period of the wave and the sample standard deviation. If the standard deviation is less than 1×10^{-4} , record the information in the data table. If not, steady the motor unit, adjust the photogate position, and repeat the trial.



² A photogate is an electronic timer containing a light beam. Every time an object blocks the light beam, the timer starts or stops.

- Record the period and the standard deviation in the data table. Keeping the same string length, adjust the motor speed until 3 nodes appear, reposition the photogate to an antinode, measure the period and measure the distance between nodes. Record your data in the table.
- Establish resonance with four nodes and five nodes and take the data as you did for 3 nodes.
- Turn off the motor.
- Turn off the LabQuest or LabPro.

DIRECTIONS FOR USING THE PHOTOGATE With LabPro

- Make sure that the TI-83 calculator and the Vernier LabPro are connected and fixed in the cradle. Plug in the LabPro and turn on the calculator.
- Connect the Photogate to the LabPro. Insert the white end of the cable into the "Dig/Sonic 1" slot on the LabPro and the other end into the opening on the photogate.
- Turn on the calculator and press "PRGM."
- Select "DATAGATE."
- Press "ENTER."
- Press "1" to go to SETUP.
- Press "3" for PENDULUM. (Do not select the PHOTOGATE option, as PENDULUM works best here.)
- Press "2" for START.
- Press "STO" when finished.

DIRECTIONS FOR USING THE PHOTOGATE With LabQuest

- Connect the power supply.
- Plug the photogate sensor into the DIG1 input of the LabQuest unit.
- Turn on the LabQuest.
- On the "File Sensors" screen, select timing.
- Set Photogate mode to Pendulum.
- Click OK.
- Press the Play/Stop button once to start the system.
- Press the Play/Stop button again to stop the system after roughly 10 seconds.
- Select Analyze at the top of the screen.
- Select Statistics from the drop down menu.
- Select the Period box that appears.
- Turn off when finished.

Data table

Overall length of string from suspension point to motor unit = _____

Number of nodes	Distance between adjacent nodes (m)	Wavelength λ in (m)	Period of vibration T in (s)	Standard deviation of period σ in (s)	Frequency f in (Hz)	Wave speed in the medium (the string) v in (m/s)
2						
3						
4						
5						

Post-lab Analysis

1. You measured the period of the oscillation and the distance between the nodes. Calculate the frequency and the wavelength for each resonance from this information, and record the results in the data table.
2. Look at the frequency column in your data table. How are the frequencies of the different resonant modes related to the frequency of the first (fundamental) mode?
3. If you were to make a standing wave with 6 nodes, predict the frequency that would produce this resonance, assuming the same length of string. Show your work.
4. Look at the wavelength column in your data table. How are the wavelengths of the different resonant modes related to the wavelength of the fundamental mode?

5. If you were to make a standing wave with 6 nodes, predict the wavelength of this resonance, assuming the same length of string. Show your work.

6. Your motor can only explore a limited range of frequencies. If you had a motor that could vibrate at any frequency, would you expect to find any additional resonant modes at lower frequencies? At higher frequencies? Explain your answer.

7. Use $v = f\lambda$ to fill in the remaining column in the data table. Use those values to make your best estimate of v , the speed of the waves on the string.

8. Does the velocity of the waves change significantly or remain essentially constant? Do you expect the velocity to change? Explain your reasoning.

9. From your values for v , predict the length of string that would resonate at 75 Hz with two nodes. Explain your reasoning.

10. You learned in this lab that the wave speed in the string stayed constant. With the help of the following equation, list two ways you could increase the wave speed of a string.

$$v = f\lambda = \sqrt{\frac{\tau}{\mu}} \quad \text{where } \tau = \text{tension of the string and } \mu = \text{mass/unit length of the string}$$